

# Number Sense

## Lesson 5: More Percents

It seems like nothing pops up more often in our "math life" than percents. We use them every day at home, at school, shopping, you name it. Let's spend a little more time with this very useful, very important topic.

How often have you seen ads like these? "15% off" or "Everything in the store is marked down 25%." One thing we should get really good at is calculating percent discounts, since we always want to save money!

So, what is the sale price of this circular saw at the right? First, we need to calculate 30% of \$150. Remember, "of" in a mathematical sentence nearly always means "times." Change 30% to its decimal equivalent, then multiply. After you calculate the discount amount, subtract it from the regular price.

$$\begin{aligned}\text{Discount: } & 30\% \text{ of } \$150 \\ & 0.30 \times \$150 = \$45\end{aligned}$$

$$\text{Sale price: } \$150 - \$45 = \$105$$

**TOOL SALE!**  
30% off

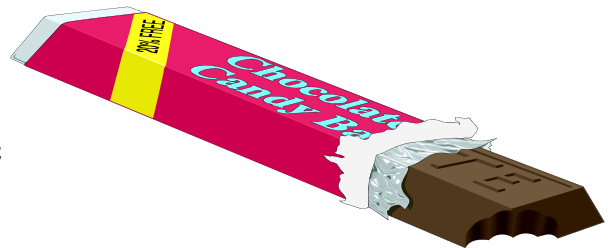


Regular Price: \$150

What about percent markup? That's how the store calculates the price you pay for an item. For example, suppose the store pays 30 cents for your favorite candy bar, and they mark candy up 50%.

$$\begin{aligned}\text{Markup: } & 50\% \text{ of } 30¢ \\ & 0.50 \times 30¢ = 15¢\end{aligned}$$

$$\text{Retail price: } 30¢ + 15¢ = 45¢$$



There's a shortcut for both percent discount and percent markup. Let's start with discount. In the example above, the circular saw has a 30% discount. This means that you will pay only 70% of the regular price ( $100\% - 30\% = 70\%$ ). So, you can simply multiply the original price by 70% to calculate the sale price.

$$\text{Sale price: } 70\% \text{ of } \$150 = 0.70 \times \$150 = \$105$$

What percent of the store's cost for the candy bar do you pay? The store marks the candy up 50%, so that means you will pay 150% of what the store pays. Multiply the store's cost by 150% to calculate the retail price.

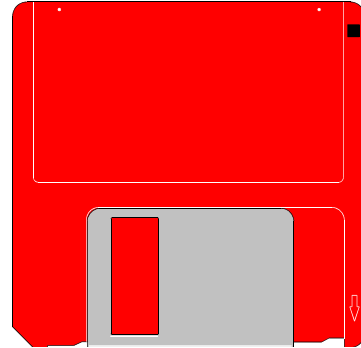
$$\text{Retail price: } 150\% \text{ of } 30¢ = 1.50 \times 30¢ = 45¢$$

In what other ways do we use percent? Often we need to determine percent from other information. For example, suppose we purchase 50 computer disks and discover that 3 of the disks are defective. What percent of the disks are defective?

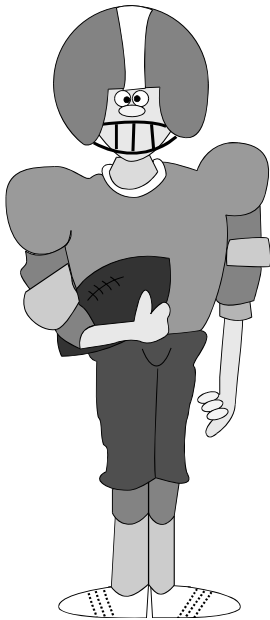
First, determine what *fraction* of the disks are defective. Three out of 50 are defective, or  $\frac{3}{50}$  of the disks. Change the fraction to a decimal, then change the decimal to a percent.

$$\frac{3}{50} = 3 \div 50 = 0.06 = 6\%$$

6% of the disks are defective.



Occasionally you need to work a percent problem "backwards." For example, suppose that you were told that "8% of the boys in the high school are on the football team." Some quick research with a program from last week's game tells you that there are 40 boys on the football team. How many boys are there in the high school?



In this case you have the equation: 8% of some number = 40. Remember we said earlier that "of" nearly always means "times." In this case, though, you can't multiply because you don't have the other number. 8% of what? You don't know! Since you can't multiply, try *dividing*.

$$\begin{aligned} 8\% \text{ of } ? &= 40 \\ 40 \div 8\% &= 40 \div 0.08 = 500 \\ 8\% \text{ of } 500 &= 40 \end{aligned}$$

There are 500 boys in the high school.

# Number Sense

## Exercise Set 5

### Part 1: General Practice

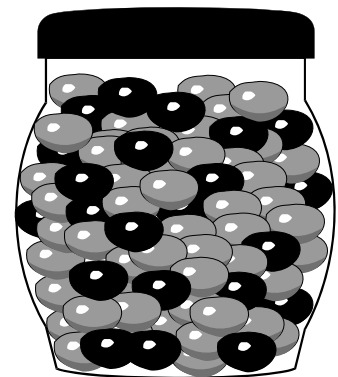
- Calculate as indicated:
  - 20% of 80
  - 5% of 400
  - 15% of 75
  - 85% of 960
  - 325% of 44
  - 0.5% of 2,400
- Calculate the sale price as indicated:
  - A \$75 jacket on sale for 40% off.
  - A \$288 stereo on sale for 25% off.
  - A \$85 tire on sale for 15% off.
- Calculate the price after markup:
  - The store pays \$140 for a TV and marks it up 65%.
  - The store pays \$6 for a CD and marks it up 150%.
  - The dealership pays \$11,800 for a car and marks it up 15%.
- The girls volleyball team won 15 of the 24 games they played. What percent of their games did they win?
- Of the 84 free throws Brad attempted during the basketball season he made 61 of them. What percent did Brad make? Round to a whole percent.
- In a recent election 40% of the voters voted for candidate Brown. Brown received 43,500 votes. How many people voted?

## Part 2: Multiple Choice Practice

1. Roger and Pat went to a restaurant for dinner. Pat ordered lasagna, which costs \$4.75. Roger is buying dinner for both himself and Pat, but he only has \$10. Which of the following can he order and still have enough money for a 15% tip?  
a) Steak dinner at \$8.50                      b) Hamburger and fries at \$5.25  
c) Lasagna at \$4.75                              d) Soup and sandwich at \$3.50
2. At one store, the price of hamburger is \$1.89 a pound for a 5-pound package. If you buy a 25-pound package, the price is 15% less. The price per pound for the 25-pound package is about  
a) \$0.28                      b) \$1.61                      c) \$1.74                      d) \$2.17
3. Suppose Martha has a job sewing zippers on down parkas. She gets 20 percent faster each day she works for the first ten days. If Martha can do 60 parkas the first day, about how many will she probably be able to handle by the end of the sixth day?  
a) About 90                      b) About 120                      c) About 150                      d) About 200
4. A refrigerator was sold for \$273, yielding a 30% profit on the cost. For how much should it be sold to yield only a 10% profit on the cost?  
a) \$210                      b) \$221                      c) \$231                      d) \$235

## Part 3: Problem-Solving Practice

For her birthday Marci received a candy jar containing 300 jelly beans. 40% of the jelly beans were black. Marci doesn't like black jelly beans, so she only ate the red ones. Two days later 60% of the jelly beans remaining in the jar were black. How many red jelly beans did Marci eat?

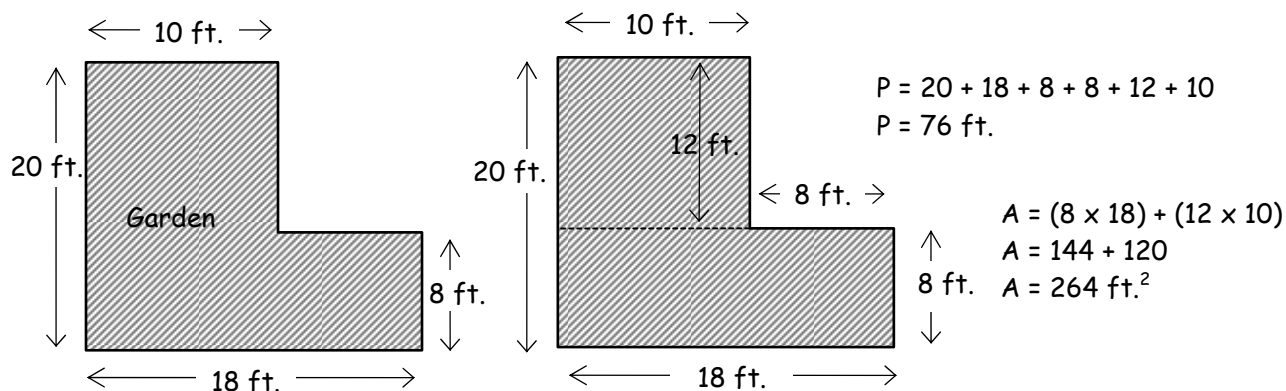


# Measurement

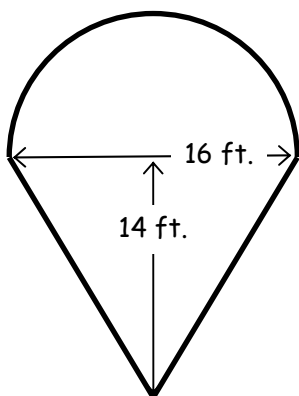
## Lesson 6: Irregular Shapes, Other Quadrilaterals

So, not everything in life is nice and neat like, say, a simple rectangle. Sometimes shapes get a little complicated and confusing. Often we need to break a complicated shape into several simple shapes before we can calculate area or perimeter.

Suppose you have a garden of the shape shown below. You plan to put a fence around it, so you need to know the perimeter, and you need to calculate its area to know how much fertilizer to buy.



First, calculate the lengths of the unmarked edges. Using the measurements of the other two vertical sides, you can calculate the unmarked vertical side as  $20 \text{ ft.} - 8 \text{ ft.} = 12 \text{ ft.}$  The unmarked horizontal side is  $18 \text{ ft.} - 10 \text{ ft.} = 8 \text{ ft.}$  Now the perimeter can be easily calculated. Finally, break the garden into two simple rectangles to calculate the area.



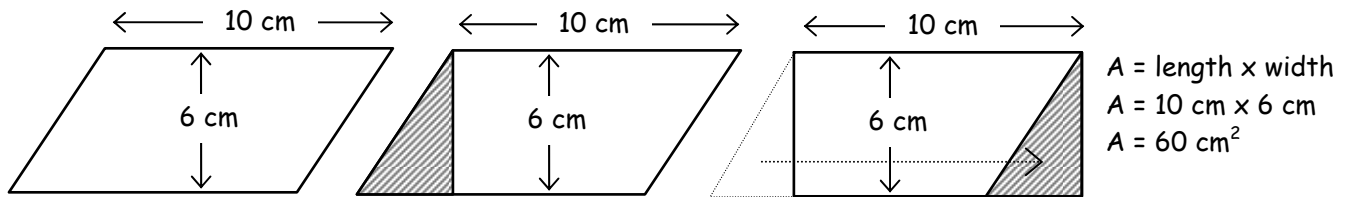
The figure at the left is painted on a playground. What is the area of this figure? To calculate this you must break it up into a triangle and half of a circle.

Triangle  
 $A = \frac{1}{2} (16 \times 14)$   
 $A = \frac{1}{2} (224)$   
 $A = 112 \text{ ft.}^2$

Half Circle  
 $A = \frac{1}{2} (\pi r^2)$   
 $A = \frac{1}{2} (3.14 \times 8^2)$   
 $A = \frac{1}{2} (3.14 \times 64)$   
 $A = \frac{1}{2} (200.96)$   
 $A = 100.48 \text{ ft.}^2$

Total Area  
 $A = 112 + 100.48$   
 $A = 212.48 \text{ ft.}^2$

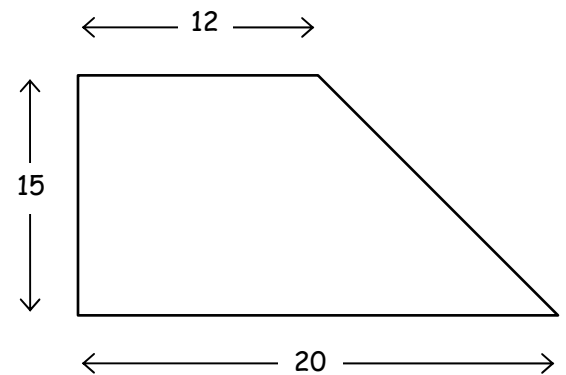
There are several more quadrilaterals that we need to look at. Let's start with the parallelogram. You'll remember that to calculate the area of a rectangle you multiply length times width. It turns out that you do exactly the same for a parallelogram. The diagram below should help you see why.



What about a trapezoid? Well, they're a little bit more complicated, but can still be calculated quite easily. A trapezoid has one pair of sides that are parallel. We call these parallel sides the bases of the trapezoid. We need to know the length of both bases, as well as the height. To calculate the area we first add the two bases, then multiply this result times the height, then divide this result by 2. The formula can be expressed:

$$A = \frac{(b_1 + b_2) \times h}{2}$$

where  $b_1$  and  $b_2$  are the two bases and  $h$  is the height.



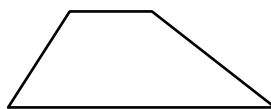
$$A = \frac{(20 + 12) \times 15}{2}$$

$$A = \frac{32 \times 15}{2}$$

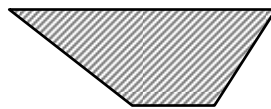
$$A = \frac{480}{2}$$

$$A = 240 \text{ square units}$$

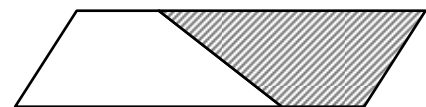
To see why this formula works, take a trapezoid, duplicate it, rotate the duplicate 180 degrees, and attach it to the original trapezoid.



Original trapezoid



Duplicate of original trapezoid rotated 180 degrees.



Original trapezoid plus duplicate.

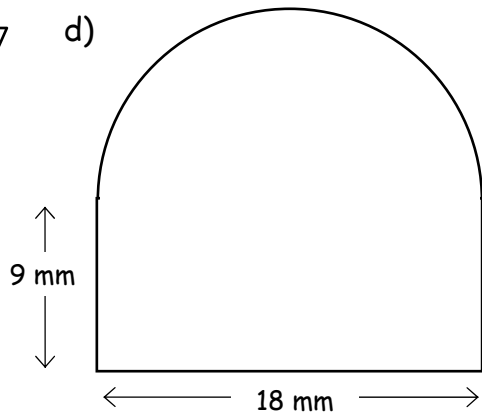
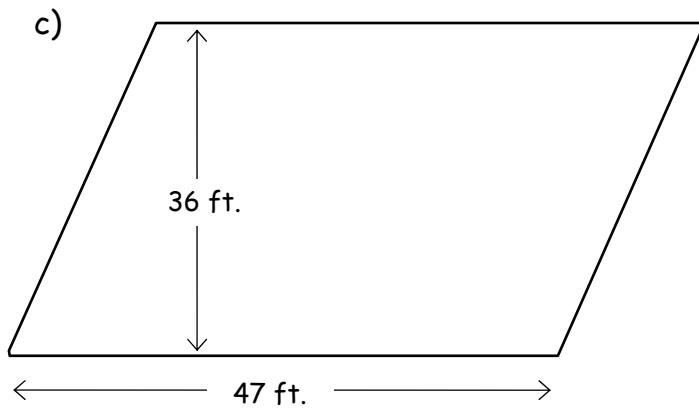
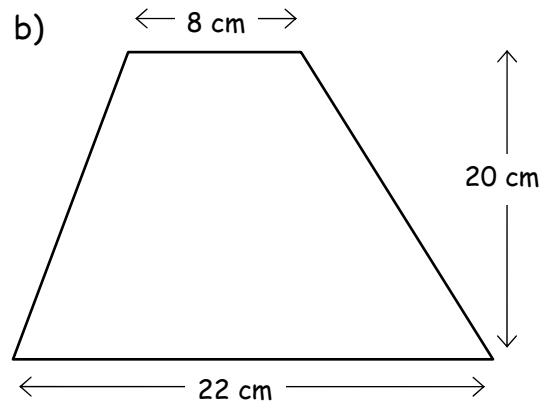
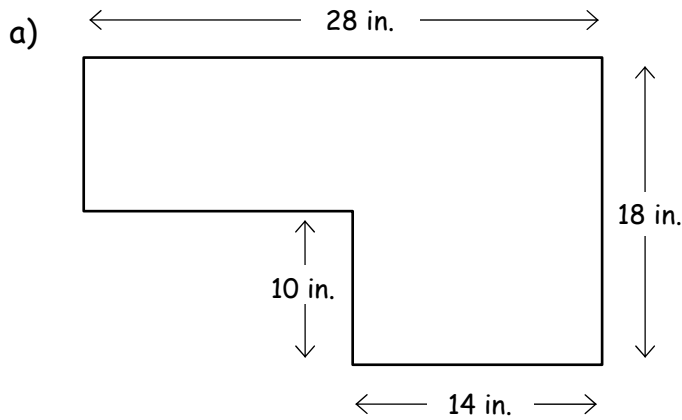
Notice that when you join the original trapezoid with the duplicate you now have a parallelogram. The length of the parallelogram is the two bases of the trapezoid added together. The height is the same as the original trapezoid. Multiply the length  $(b_1 + b_2)$  times the height to get the area of the parallelogram. However, this is double the area of the original trapezoid, so divide this result by 2.

# Measurement

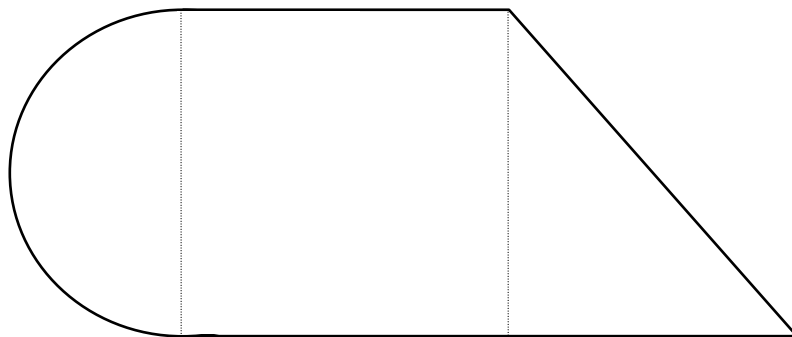
## Exercise Set 6

### Part 1: General Practice

1. Calculate the area of each of the following figures:

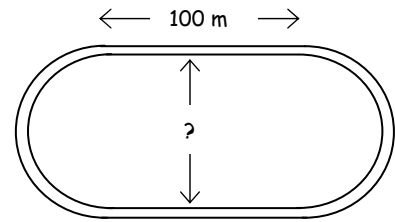


2. Calculate the area and perimeter of the following figure. The center of the figure is a square measuring 20 inches on each side. The left side is a half circle, and the right side is a triangle with a 15 inch base.



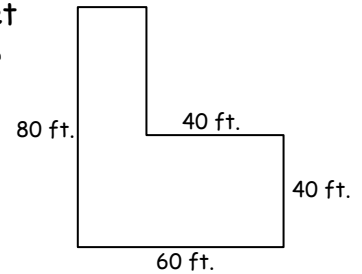
Part 2: Multiple Choice Practice

1. The jogging track shown at the right is 400 meters around. The straight sections are 100 meters. About how far apart are the straight sections?



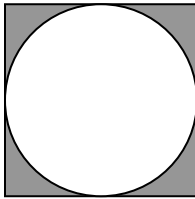
- a) 50 m    b) 64 m    c) 72 m    d) 100 m

2. If a 20-pound bag of fertilizer will cover 1,000 square feet of lawn, how many pounds of fertilizer will it take to cover the lawn shown at the right?



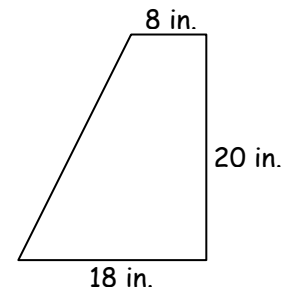
- a) 10 pounds    b) 64 pounds    c) 80 pounds    d) 96 pounds

3. What percent of the figure below is shaded?



- a) 30%  
b) 24.8%  
c) 21.5%  
d) 20%

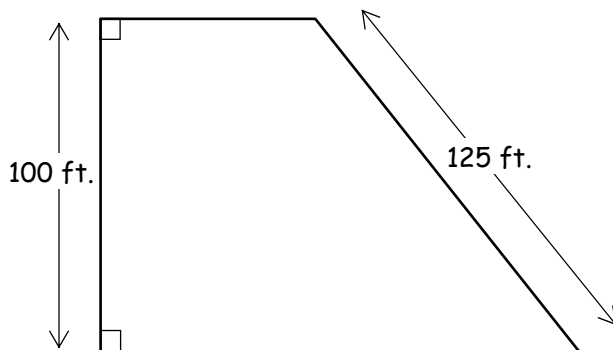
4. What is the area of the figure at the right?



- a) 260 sq. in.    b) 360 sq. in.    c) 160 sq. in.    d) 46 sq. in.

Part 3: Problem-Solving Practice

A garden has the shape and dimensions shown. If it can be enclosed in 400 feet of fence, what is the area of the garden?



# Algebraic Sense

## Lesson 6: Linear Equations

Linear equations are used a lot in algebra, and we are frequently asked to graph them. In the last lesson we discovered that if we develop a table of ordered pair solutions to a linear equation, then plot those ordered pairs, the points representing those ordered pairs all line up. A line drawn through the points represents *all* of the solutions to the equation.

Is there a quicker method of determining the graph of a linear equation? Would I ask the question if there wasn't? Of course there is! A linear equation is usually expressed in slope-intercept form:  $y = mx + b$ .  $m$  tells us the slope of the line, and  $b$  tells us the y intercept.

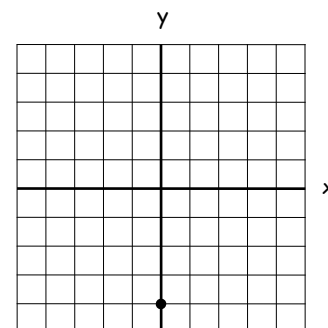
Slope-Intercept Form:

$$y = mx + b$$

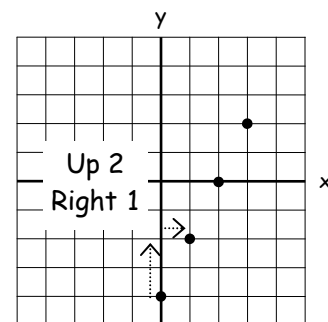
$m$ : slope

$b$ : y intercept

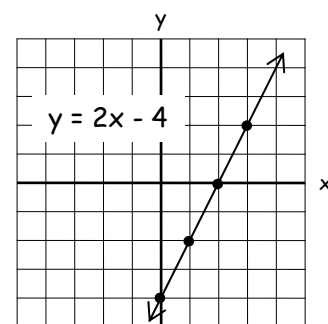
Let's again consider the equation  $y = 2x - 4$ . Since this equation is in slope-intercept form, all we need to do is look at the  $m$  and  $b$  values. The  $b$  value tells us where the line will cross the y axis. In this case it will cross the y axis at  $-4$ , so we'll put a dot there.



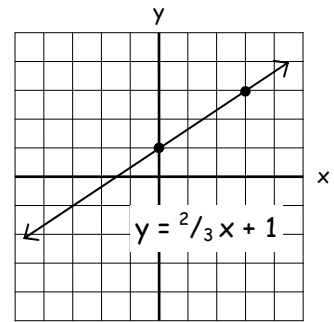
Next, we look at the  $m$  value, which tells us the slope. Slope is defined as rise over run. We think of it as a fraction where the numerator is the amount of vertical change (rise), and the denominator is the amount of horizontal change (run). In this example  $m$  is 2, which expressed as a fraction is  $2/1$ . This means that for every 2 units of vertical change there is 1 unit of horizontal change. So, start at the dot we placed on the y axis, go up 2 and right 1, and place another dot. Now, start at this dot, go up 2 and right 1, and place another dot. Do this several times to establish a number of points on the line.



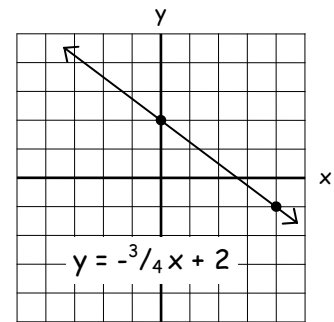
Finally, draw a line through the points. This line is the graph of the equation  $y = 2x - 4$ . The  $x$  and  $y$  values of any point on this line are an ordered pair which is a solution to the equation.



Often the coefficient of  $x$ , the  $m$  value, is a fraction. This is actually quite handy. Since  $m$  is the slope, which is defined as rise over run, a fraction value of  $m$  is easy to use. The numerator is the rise, and the denominator is the run. Consider the equation  $y = \frac{2}{3}x + 1$ . First, the  $b$  value tells us that the line will cross the  $y$  axis at  $+1$ , so plot a point there. From this point go up 2 (rise), and right 3 (run). Connect these points with a line to complete the graph.



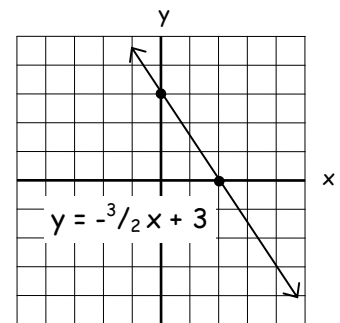
What if the  $m$  value, is negative? Consider the equation  $y = -\frac{3}{4}x + 2$ . Remember,  $m$  is the slope, which is rise over run. In this case the rise is  $-3$  and the run is  $4$ . First, plot the  $y$  intercept at  $+2$ . From there move down 3 (a *negative* rise is down rather than up), then right 4. Connect these points with a line to complete the graph.



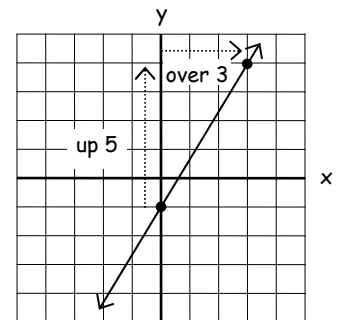
If the equation is not in slope-intercept form, we'll need to use some basic algebra to rearrange it. Consider the equation  $2y + 3x = 6$ . Before we can graph this equation it must be changed to the form  $y = mx + b$ .

Initial equation, not in slope-intercept form:	$2y + 3x = 6$
Subtract $3x$ from both sides:	$-3x \quad -3x$
Divide both sides by 2:	$\frac{2y}{2} = \frac{-3x + 6}{2}$
(note right side: $-3x \div 2$ and $6 \div 2$ )	
Now in slope-intercept form:	$y = -\frac{3}{2}x + 3$

Now that we have the equation in slope-intercept form, we can use the  $m$  and  $b$  values to graph it. Plot the  $y$  intercept at  $+3$  on the  $y$  axis. From there, go down 3 (negative slope) and right 2, and plot another point. Connect these points with a line to complete the graph.



Sometimes we are given the graph and asked to determine the equation. To do this we just work "backwards." The graph at the right crosses the  $y$  axis at  $-1$  so we know  $b$  is  $-1$ . Find another easily identified point on the line, then count up (rise) and over (run) from the  $y$  intercept to this point. Slope is rise over run, so the slope of this line is  $\frac{5}{3}$ . The complete equation is  $y = \frac{5}{3}x - 1$ .



# Algebraic Sense

## Exercise Set 6

### Part 1: General Practice

1. Change each of the following equations to slope-intercept form:

a)  $y - 2x = 5$

b)  $3x + 2y = -8$

c)  $15 + 3y = 4x$

2. Identify the slope and the y-intercept of each of the following equations:

a)  $y = 3x - 7$

b)  $y = -2x + 4$

c)  $y = -\frac{5}{3}x - 2$

3. Slope is defined as "rise over run." What is the rise and run of each of the following equations:

a)  $y = 5x + 9$

b)  $y = -\frac{4}{7}x + 4$

c)  $y = \frac{9}{2}x - 5$

rise:      run:

rise:      run:

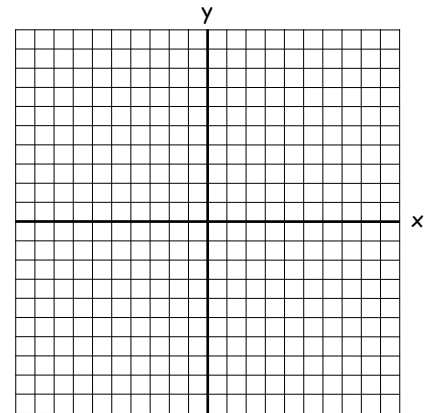
rise:      run:

4. Graph each of the following equations:

a)  $y = \frac{2}{3}x - 5$

b)  $y = -2x + 3$

c)  $4y - 5x = -16$

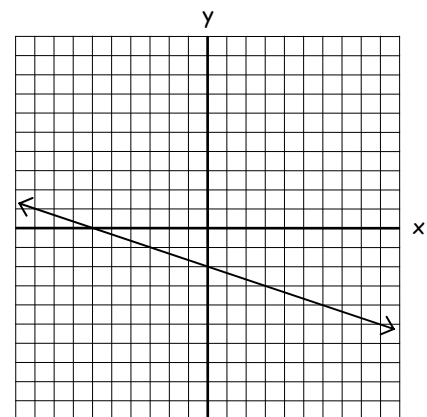


5. From the graph shown determine the:

a) y-intercept.

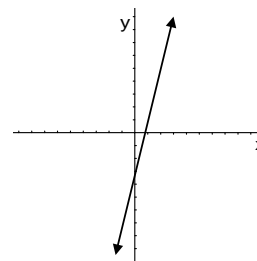
b) slope.

c) equation of the line.

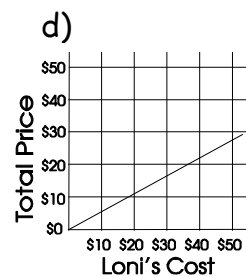
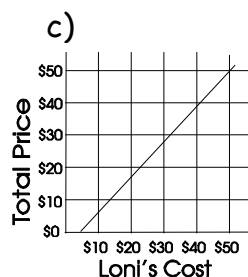
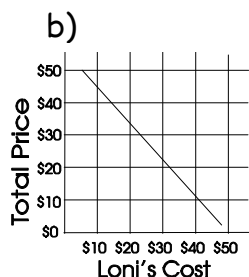
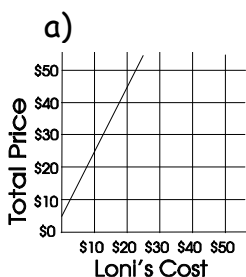


## Part 2: Multiple Choice Practice

1. Only one of the four equations below could possibly be the equation for the graph at the right. Which one is it?
- a)  $y = 2x + 3$       b)  $y = 2x - 3$   
 c)  $y = -3x - 2$       d)  $y = 3x + 2$



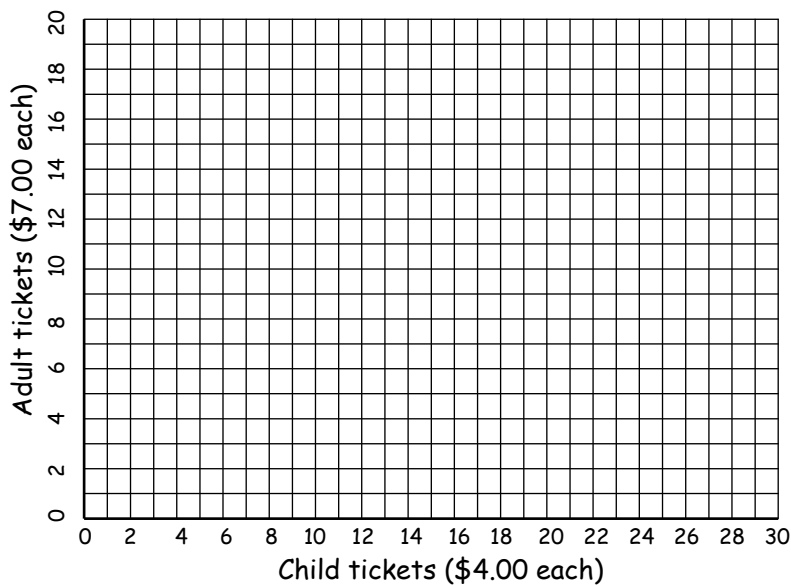
2. Loni is opening a mail order house. For the clothes she sells, she wants to charge twice what they cost her plus a \$5.00 shipping and handling charge. Which chart shows her pricing plan?



3. The equation  $4y + 3x = 12$  changed to slope-intercept form is
- a)  $y = \frac{3}{4}x + 3$       b)  $y = -\frac{3}{4}x + 3$       c)  $y = -\frac{3}{4}x + 12$       d)  $x = -\frac{4}{3}y + 3$
4. The graph of the equation  $3x - 5y = 15$  will cross the y axis at
- a) 3      b) -3      c) 5      d) -5

## Part 3: Problem-Solving Practice

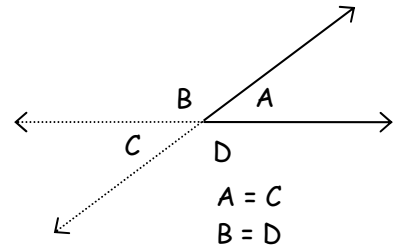
Admission to a museum is \$4.00 per child and \$7.00 per adult. A group of people paid \$105.00 for admission. Using  $x$  for children and  $y$  for adults, construct the linear equation that describes the information in this problem, then graph the equation. Identify the points on the graph that represents all the possible combinations of children and adults that could be in this group.



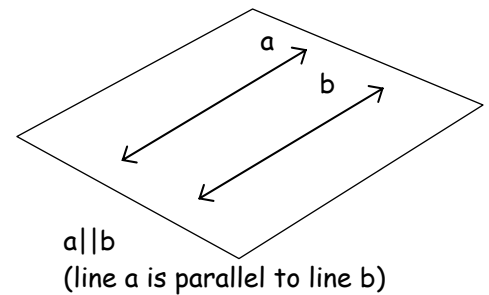
# Geometric Sense

## Lesson 2: More About Lines and Angles

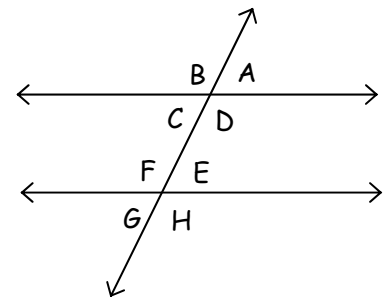
We know from the last lesson that if two rays share the same vertex they create an angle, and we can measure the angle with a protractor. Well, let's extend those rays into lines and take a look at the new angles we've created. When two lines intersect, they create four angles, angles A, B, C, and D in the diagram at the right. The two angles on opposite sides of the vertex are called vertical angles. Vertical angles are equal in measure.



If two lines are on the same plane, in most cases these lines will "cross" or intersect somewhere on the plane. However, it is possible for two lines in the same plane to never intersect. We call these parallel lines.



If two parallel lines are cut by a third line, there are some important things about the resulting angles we should know. Consider the diagram at the right. We already know that  $A = C$ ,  $B = D$ ,  $E = G$ , and  $F = H$ , because these are all examples of vertical angles. But, there's more. Notice that angles A, B, C, and D look very similar to angles E, F, G, and H. Angles in corresponding regions of these "clusters" of angles are called corresponding angles and are equal. In this diagram A and E are corresponding, so  $A = E$ . Also,  $B = F$ ,  $C = G$ , and  $D = H$ .



But, there's still more. Since  $A = C$  and  $A = E$ , then  $C = E$ . We call these alternate interior angles because they're on the interior of the two parallel lines, and on alternate sides of the third line. Alternate interior angles are equal. For similar reasons,  $B = H$ . These are called alternate exterior angles, and are also equal.

Vertical Angles:

$$A = C, B = D, E = G, F = H$$

Corresponding Angles:

$$A = E, B = F, C = G, D = H$$

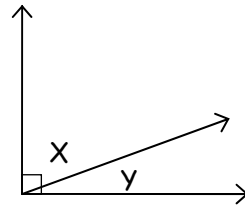
Alternate Interior Angles:

$$C = E, D = F$$

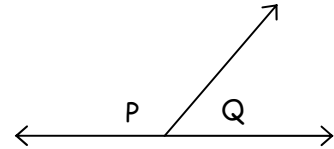
Alternate Exterior Angles:

$$A = G, B = H$$

If two angles add up to 90 degrees, they are complementary angles. In the diagram at the right, angles X and Y are complementary. If we know the measure of one of these angles we can calculate the other. For example, if  $X = 70$  degrees, then  $Y = 20$  degrees ( $90 - 70$ ).

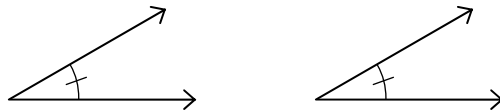


If two angles add up to 180 degrees they are supplementary angles. In the diagram at the right, angles P and Q are supplementary, and if we know the measure of one we can calculate the other. For example, if  $P = 115$  degrees, then  $Q = 65$  degrees ( $180 - 115$ ).



In all of the examples above, when two angles have the same measure, we say they are congruent. The term *congruent* means "the same," and it's a term we use a lot in geometry. In this case it means two angles have the same *degree measure*. If we refer to two line segments as congruent, we mean they have the same *length*.

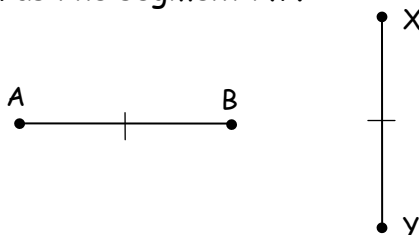
The two angles below are congruent. Note the use of "tick" marks to indicate that the angles are the same measure.



If a figure contains more than one set of congruent angles they will be marked with increasing numbers of tick marks, as shown below.



Two line segments that are congruent can also have tick marks. Line segment AB below is the same length as line segment XY.



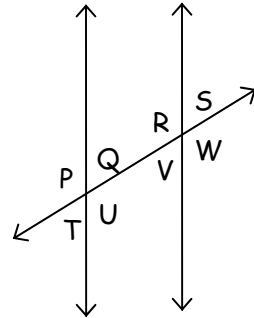
# Geometric Sense

## Exercise Set 2

### Part 1: General Practice

1. In the diagram at the right...

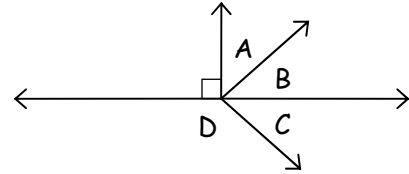
- angles P and R are examples of \_\_\_\_\_ angles.
- angles S and V are examples of \_\_\_\_\_ angles.
- angles S and T are examples of \_\_\_\_\_ angles.
- angles R and U are examples of \_\_\_\_\_ angles.



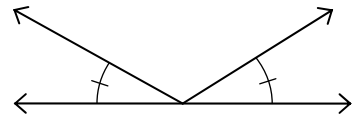
2. List all of the angles in the diagram above that are equal to angle Q, and why.

3. In the diagram at the right...

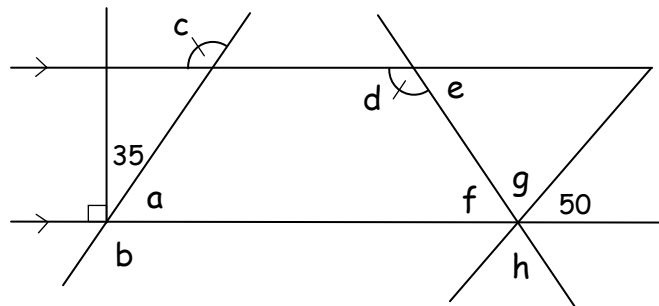
- which two angles are supplementary?
- which two angles are complementary?



4. In the diagram at the right, what do the arcs with the small "tick" marks indicate?



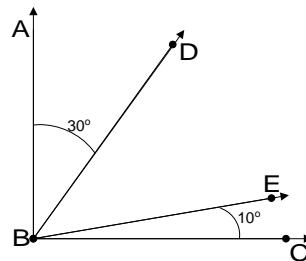
5. The diagram below indicates the measurement of two angles, one 35 degrees and the other 50 degrees. With this information and everything else you've covered in this lesson, calculate the measurement of the lettered angles. (The small sideways v-shaped marks indicate parallel lines.)



Part 2: Multiple Choice Practice

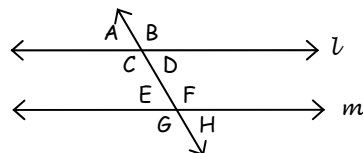
1. Given that  $\angle ABC$  is a right angle, what is the measure of  $\angle DBE$ ?

- a)  $5^\circ$       b)  $20^\circ$       c)  $50^\circ$       d)  $140^\circ$



2. Given that line  $l$  is parallel to line  $m$ , which of the following are true?

- a) Angles A and C are vertical angles  
 b) Angles A and H are alternate exterior angles  
 c) Angles A and F are alternate interior angles  
 d) Angles A and G are corresponding angles



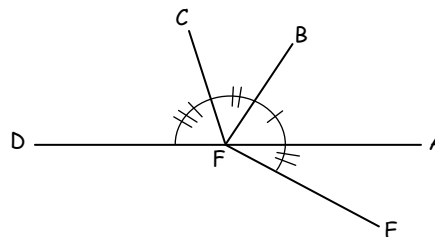
3. Angles P and Q are



- a) congruent      b) corresponding      c) complementary      d) supplementary

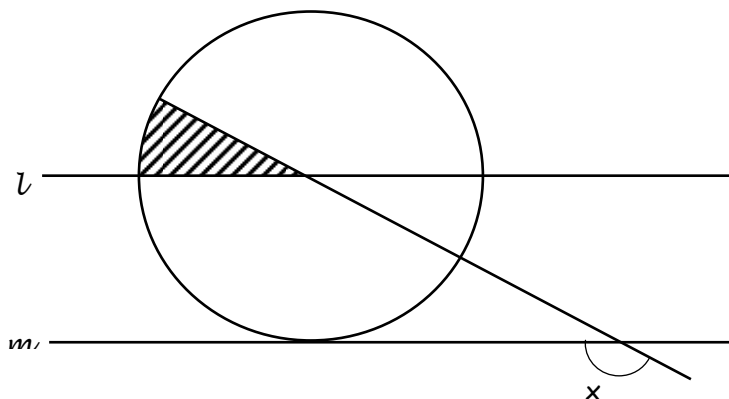
4. The diagram at the right indicates

- a)  $\angle AFB$  and  $\angle AFE$  are complementary  
 b)  $\angle CFD$  and  $\angle AFB$  are congruent  
 c)  $\angle CFB$  and  $\angle AFE$  are congruent  
 d)  $\angle DFB$  and  $\angle BFE$  are supplementary



Part 3: Problem-Solving

Parallel lines  $l$  and  $m$  are 20 mm apart. The area of the shaded region is  $100 \text{ mm}^2$ . To the nearest degree, what is the measure of angle  $x$ ?



# Probability & Statistics

## Lesson 1: Probability

When *George* gets up in the morning it's still dark. While getting dressed he reaches into his sock drawer where there are 6 pair of blue socks, 8 pair of brown socks, and 10 pair of black socks. It's too dark for him to tell one color from another, and the socks are mixed up in the drawer. What is the probability that *George* will randomly select a blue pair? (*George* could simply turn the light on to improve his chances to 100%, but then we wouldn't have such a fun problem to work on!)

To determine the probability of a random event happening, all we need to do is make a fraction. The numerator is described technically as the number of favorable outcomes. Selecting blue socks is the desired, or *favorable* outcome in this case, and there are 6 pair of blue socks, so our numerator is 6. The denominator is the total number of outcomes, in our case 24, the total number of socks in the drawer.

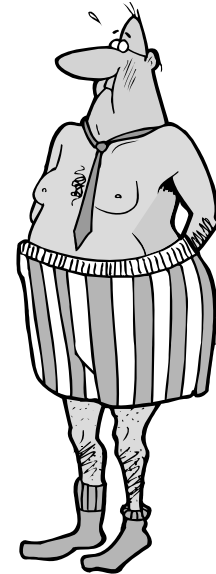
$$\begin{array}{l} \text{Blue socks in drawer: } \frac{6}{24} = 0.25 \text{ or } 25\% \\ \text{Total socks in drawer: } \end{array}$$

Change the fraction to a decimal value or percent to describe the probability. There is a 25% probability that *George* will select the blue socks.

Now, what's the probability of *George* selecting blue socks again? Well, this depends on whether he puts the first pair back. If he puts the first pair back, then we describe these two events as independent. In other words, nothing about the outcome of the first event has any effect on the outcome of the second event. The probability of *George* selecting blue socks the second time after putting the first pair back is again 25%.

But, if *George* keeps his first pair of socks, this changes things. The probability of the second event becomes dependent on the outcome of the first event. If we assume that he successfully selected blue socks the first time, there are now only 5 pair of blue socks in the drawer, and there are only 23 pair of socks left in the drawer. Now the probability of selecting a blue pair is  $\frac{5}{23}$ .  $5 \div 23 = 0.217$  (rounded), so the probability of *George* selecting blue socks the second time after keeping the first pair, assuming the first pair was blue, is about 21.7%.

What's the probability that *George* will select two blue pair in a row? The probability of selecting a blue pair twice is the probability of selecting a blue pair the first time *times* the probability of selecting a blue pair the second time. Well, again this depends on whether he puts his first pair back or not. If he puts the first pair

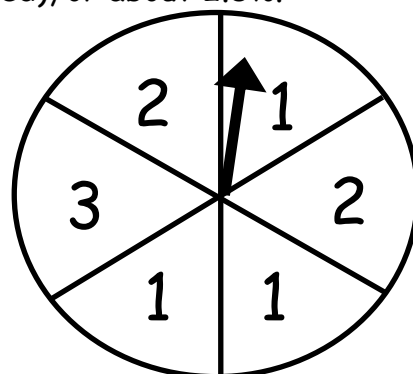


back, then the probability is  $\frac{6}{24} \times \frac{6}{24} = \frac{36}{576} = 0.0625$ , or 6.25%. If he keeps the first pair out, then the probability is  $\frac{6}{24} \times \frac{5}{23} = \frac{30}{552} = 0.0543$  (rounded), or about 5.43%.



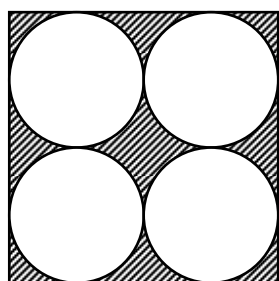
Let's apply what George has taught us to some other situations. What is the probability of rolling a 3 with a single die? A die has 6 sides, so 6 total outcomes, and only one of them is a 3, so the probability is  $\frac{1}{6} = 0.167$  (rounded), or about 16.7%. What's the probability of rolling a pair of 3's with two dice? Well, it's  $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36} = 0.028$  (rounded), or about 2.8%.

Consider this game spinner. What is the probability of spinning each of the numbers? There are 6 possible outcomes. 3 outcomes are 1, 2 outcomes are 2, and 1 outcome is 3.



- Spin a 1:  $\frac{3}{6} = 0.5 = 50\%$
- Spin a 2:  $\frac{2}{6} = 0.333$  (rounded) = 33.3%
- Spin a 3:  $\frac{1}{6} = 0.167$  (rounded) = 16.7%

← 4 ft. →



If a dart is thrown randomly at the dart board at the left, what is the probability that it will land in the shaded area? To determine this we must first calculate the area of the entire board and the area of the four circles. The board's area is 16 sq. ft. ( $A = 4 \times 4$ ). The radius of each circle is 1 ft., so the area of one circle is 3.14 sq. ft. ( $A = \pi r^2 = 3.14 \times 1^2$ ). The area of all four circles is 12.56 sq. ft. ( $4 \times 3.14$ ), so the shaded area is 3.44 sq. ft. ( $16 - 12.56$ ). The probability, then, is determined by making a fraction of the two areas.  $\frac{3.44}{16} = 0.215$  or 21.5%.

What about the probability of picking the three winning lottery numbers, each selected from the numbers 1 through 20? Well, this gets a bit complicated, but we can do it! On the first pick there are 3 favorable outcomes (numbers you hope are picked) out of 20, for a probability of  $\frac{3}{20}$ . Assuming one of your numbers is picked the first time, on the second pick there are 2 favorable outcomes out of 19, for a probability of  $\frac{2}{19}$ . Assuming two of your numbers are picked the first two times, then on the last pick there is 1 favorable outcome out of 18, for a probability of  $\frac{1}{18}$ . The probability of all three of your numbers being picked is  $\frac{3}{20} \times \frac{2}{19} \times \frac{1}{18} = \frac{6}{6840} = \frac{1}{1140}$ . You only have 1 chance in 1,140, or a 0.09% chance! In many state lotteries you pick 6 numbers from the numbers 1 through 40. Your chances of winning are  $\frac{6}{40} \times \frac{5}{39} \times \frac{4}{38} \times \frac{3}{37} \times \frac{2}{36} \times \frac{1}{35} = \frac{720}{2,763,633,600}$ . This is equal to 1 chance in 3,838,380, or a 0.0000261% chance!

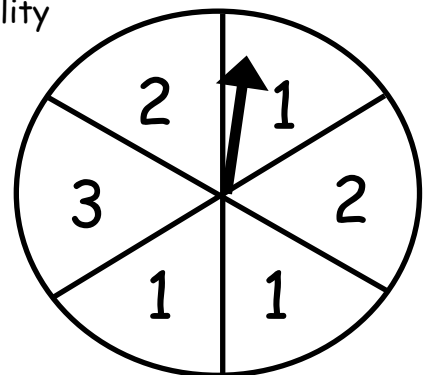
# Probability & Statistics

## Exercise Set 1

### Part 1: General Practice

1. What is a *favorable outcome*?
2. Describe the fraction used to determine the probability of an event.
3. There are 6 red marbles, 4 white marbles, and 2 blue marbles in a bag. What is the probability of randomly selecting...
  - a) a red marble?
  - b) a white marble?
  - c) a blue marble?
  - d) a white marble then put it back in the bag, then selecting a white marble again?
  - e) a white marble and keeping it out of the bag, then selecting a white marble again?
  - f) a red marble followed by a white marble followed by a blue marble, without returning any marbles to the bag?
4. A popular dice game uses five dice. A very high score is awarded to rolling five of the same number. What is the probability of having all five dice be the same number with just one roll of the dice?

5. a) With the spinner at the right, what is the probability of spinning a 1 or 2 in one spin?

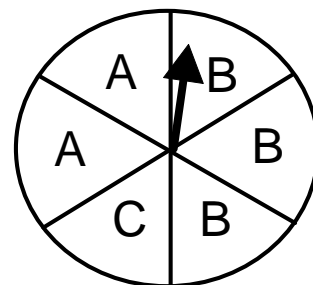


- b) What is the probability of spinning two times and having the sum of the two spins be 5 or more?

## Part 2: Multiple Choice Practice

- To win the big prize in a lottery, you have to decide which six numbers from 1 to 52 will be randomly selected. Of the three people below, which one has the greatest chance of winning the prize?
  - ALL THREE PEOPLE have exactly the same chance of winning.
  - Kenneth, who picks 1, 2, 3, 4, 5 and 6
  - Kathleen, who picks 22, 24, 26, 28, 30 and 32
  - Kyle, who picks 11, 19, 26, 34, 41 and 47
- Twenty-six squares of paper lettered A to Z are placed in a can. One letter is randomly chosen from the can and turns out to be the letter G. If a second letter is drawn without putting the letter G back into the can, what is the probability that it will be the letter F?
  - $\frac{2}{25}$
  - $\frac{2}{26}$
  - $\frac{1}{25}$
  - $\frac{1}{26}$
- In a group of 1000 adults, 682 are women. What is the probability that a person chosen at random from this group will be a man?
  - 0.318
  - 0.682
  - 0.5
  - 0.11

- What is the probability of getting an A using this spinner?
  - $\frac{2}{4}$
  - $\frac{1}{3}$
  - $\frac{1}{6}$
  - 2



## Part 3: Problem-Solving Practice

A game of chance uses the spinner shown. Would you expect to be winning or losing after 100 spins, and by how much?

